TRENT2D, a quasi-two-phase numerical code to simulate debris flow dynamics

M. Marchelli, M. Pirulli & C. Scavia
Politecnico di Torino, Torino, Italy

G. Rosatti
Università degli Studi di Trento, Trento, Italy

ABSTRACT: Debris flows are multiphase, gravity-driven flows consisting of randomly dispersed interacting phases. Their rheology and flow behaviour can vary along the runout path and depend on the sediment composition and percentage of solid and fluid phases. In an attempt to better comprehend the mechanisms and dynamics of these complex phenomena, existing one-phase numerical models are being complemented or replaced by new two-phase models. In the present paper, capabilities and limits of a quasi-two-phase code (TRENT2D) to simulate debris flows are discussed with reference to the Pellaud catchment basin (Norwestern Italy). Since TRENT2D is based on Takahashi’s theory (1978), it restricts the angles of failing slopes to less than $\phi/2$, that is most commonly exceeded at site. As a consequence, the flow dynamics in the Pellaud upper slope was simulated with a one-phase code (RASH$^{3D}$) and its outputs were used as input in the TRENT2D code, as soon as the Takahashi’s conditions are respected.

1 INTRODUCTION

In alpine small basin, debris flows are one of the most devastating landslide phenomena, in terms of loss of life and damages to structures and infrastructures. Debris flows are a mixture of non-plastic coarse material and water that flows in a steep channel (Hungr et al. 2001). Their destructive potential is due to the absence of premonitory signs, the extremely high velocity (0.05–20 m/s), the erosive capability and the long travel distance also on low-inclination slopes. Even if a comprehensive and satisfactory description of the interaction between the solid and fluid phases is not completely understood, it plays a key role in the comprehension and modelling of this flow dynamics.

In the last decades, continuum-mechanics numerical models to investigate these flows dynamics on a complex three-dimensional topography have been widely developed (e.g. O’Brien et al. 1993, Iverson & Delinger 2001, McDougall & Hungr, 2005). On the basis of the number of phases considered it can be distinguished in one-phase and two-phase models, generally neglecting the air in the interstices and assuming a completely saturated material. The aim of the two-phase modelling is to describe the flow dynamics in a more accurate way, at the expense of an easier simulation of a mixture (Iverson 1997, Pudasaini et al. 2005, Armanini et al. 2009).

When the triggering zone concerns very steep slopes that progressively change to gently sloping, debris flow could be considered as an evolutionary phenomenon: in the upper part, in proximity of the feed basin, the flow can generally be unsaturated, while in the lower part it becomes saturated.

Considering the infinite slope of homogeneous, cohesionless, isotropic soil model for debris flow, together with the assumption of slope-parallel groundwater level and a surface-water surcharge, Takahashi (1978) restricts the angle of failing saturated slopes to less than $\phi/2$, with $\phi$ the frictional angle of the soil. Moreover Takahashi’s theory (1978, 2007, 2009) focused only on a model of debris flow as a water-saturated inertial grain flow governed by Bagnold’s (1954) concept of dispersive stress, considering that the main characteristics of a debris flow would be produced by frequent collisions between coarse particles. This model could be applied in relatively gentle sloping, sediment-choked channels (Takahashi, 1991) and it disregards the possible role of liquefaction as mobilization process (Iverson et al., 1997). Nevertheless, considering also the unsaturated flow, angles of failing slopes increase, as shown at site.

From this perspective two different modelling schemes have to be adopted: in the upper part, where the flow is characterized by a mixture of debris, water and air, the one-phase model allows to overcome the problem of simulation of air
filling the voids in an unsaturated flow, considering a mixture with the characteristic of a multiphase flow. In the lower part, a two-phase model could better simulate the moving mass, only constituted by debris and water.

To the Authors knowledge, a numerical code which allows simulating the transition from unsaturated to saturated condition during the flow process does not today exist. This is because a combination of a one-phase, RASH\textsuperscript{3D} (Pirulli 2005, Pirulli et al. 2007), and a quasi-two-phase code, TRENT2D (Armanini et al. 2009), has been adopted in the following to simulate debris flows dynamics in the Pellaud catchment basin (Valle d’Aosta Region, Northwestern Italy).

The outputs of RASH\textsuperscript{3D} have been used as inputs in TRENT2D, as soon as the Takahashi’s conditions are matched. Capabilities and limits of the two methods are discussed in the following sections.

2 CONTINUUM MECHANICS MODELLING

Continuum mechanics models the behaviour of materials as continuous mass rather than as discrete particles. A heterogeneous and multiphase flowing mass, like debris flows, is then replaced by a single continuum that describes the bulking behaviour of the real heterogeneous mass, which is made of water, air, solid, if a one-phase model is assumed; while, two continuums, that is a solid and fluid phases, are considered if a two-phase model is adopted. The dynamic of a one-phase continuum equivalent fluid can be described by the following mass and momentum conservation laws:

\[ \nabla \cdot \mathbf{v} = 0 \]  \hspace{1cm} (1)

\[ \rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = \nabla \cdot \mathbf{\sigma} + \rho \mathbf{g} \]  \hspace{1cm} (2)

where \( \mathbf{v}(x,y,z,t) \) and \( \mathbf{\sigma}(x,y,z,t) \) are respectively the three-dimensional velocity vector and Cauchy stress tensor in a \((x, y, z)\) reference system, \( \rho \) is the mass density and \( \mathbf{g}(x,y,z) \) is the gravitational acceleration vector in the above mentioned reference system.

In the two-phase approach, the mathematical model is composed by the mass and the momentum conservation equation of each component. The equations are similar to Equation (1) and (2) but, in this case, a variable density (function of the phase concentration) must be considered.

Furthermore two constitutive laws for both solid and fluid phases have to be considered, together with an interaction law.

2.1 Depth averaged equations

Assuming that during debris flow motion the thickness of the moving mass is much smaller than its length, it is possible to integrate Equations (1)–(2) in depth, obtaining the so-called depth averaged continuum flows model (Savage & Hutter 1989).

With this assumption any change in the mechanical behaviour within the flux is neglected and the constitutive equations of the moving mass are incorporated into a single term which describes the stress at the interface between the flowing material and the surface of the bed path. Nevertheless in the two-phase model this stress term is subdivided into two parts, one for the solid and one for the liquid phases.

Both RASH\textsuperscript{3D} and TRENT2D start from this assumption, for each of the considered phases.

3 RASH\textsuperscript{3D} VS TRENT2D

RASH\textsuperscript{3D} and TRENT2D, are both based on depth-averaged equations, assume isotropy of normal stresses and adopt a finite volume scheme for the numerical implementation of balance equations, but they differ in terms of number of phases considered, rheological laws, possible entrainment and numerical expression of fluxes.

Both the codes need the pre-event Digital Elevation Model (DEM) of the study area as input data and the definition of the position of the source area of the debris flow as well as the magnitude of the triggered mass. As far as this aspect is concerned, the two codes are different. In RASH\textsuperscript{3D} the boundary of the source area, the geometry of the initial volume and the rheological parameters of the mass must be specified. In TRENT2D an inflow flood hydrograph of the mixture is required, together with the local slope of the area of the initial section of the flow.

Both rheological and physical parameters required in the two codes are examined in detail in the further sections.

3.1 RASH\textsuperscript{3D} model

The RASH\textsuperscript{3D} code (Pirulli 2005) is an upgrade of a previously-existing numerical code (SHWCIN) (Audusse et al. 2000, Mangeney et al. 2003) developed by INRIA (Institut National de Recherche en Informatique et en Automatique, France) and IPGP (Institut de Physique du Globe de Paris, France).

In RASH\textsuperscript{3D} the continuity Equation (1) and the momentum Equation (2) are depth-averaged, in a reference system having the z direction aligned with the local bed normal direction, and expressed as follows:
\[ \frac{\partial h}{\partial t} + \frac{\partial (hv_x)}{\partial x} + \frac{\partial (hv_y)}{\partial y} = 0 \]  

(3)

\[ \frac{\partial (hv_x)}{\partial t} + \frac{\partial (hv_x^2)}{\partial x} + \frac{\partial (hv_xv_y)}{\partial y} + \frac{\partial g}{\partial x} \left( \frac{h^2}{2} \right) = 0 \]  

(4)

\[ \frac{\partial (hv_y)}{\partial t} + \frac{\partial (hv_xv_y)}{\partial x} + \frac{\partial (hv_y^2)}{\partial y} + \frac{\partial g}{\partial y} \left( \frac{h^2}{2} \right) = 0 \]  

(5)

where \( h \) is the flow depth measured along the z-direction, \( v_x, v_y \), the depth-averaged components of the velocity vector \( \bar{v} \), \( S_{f_i} \) and \( S_{f_s} \) are the friction slope components. The solution has an open rheological kernel, so that the variety of rheologies can be implemented. Since the Voellmy rheology is considered the more suitable for debris flow simulation by many Authors (e.g. Pirulli & Sorbino 2008), it has been used in the following simulations. The Voellmy rheology consists of a Coulomb or basal friction term (Rickenmann & Koch, 1997), and a turbulent term, which accounts for velocity-dependent friction stresses.

\[ S_{f_{i=x,y}} = \cos \alpha \tan \phi + \frac{v_y^2}{h \xi} \]  

(6)

where \( \alpha \) = inclination of the bed from horizontal, \( \phi \) = basal friction angle, \( \xi \) = turbulence coefficient.

The number of the unknowns, so coupling the Equation (3), (4), (5) with the dynamic condition at the bottom (6), the system can be solved, in terms of flow depth \( h \) and flow velocity components \( v_x \) and \( v_y \), thus neglecting the interphase drag. Anyhow, the mobile-bed approach, considering both erosion and deposition phenomena, allows the description of segregation of sediments respect-water in the stopping phase of the flow.

The depth-averaged partial differential equations, in a Cartesian coordinate system \((x, y, z)\), with \( x, y \)-axes horizontal and \( z \)-axis upward vertically oriented, are:

\[ \frac{\partial (z_h + h)}{\partial t} + \frac{\partial (hv_x)}{\partial x} + \frac{\partial (hv_y)}{\partial y} = 0 \]  

(7)

\[ \frac{\partial (c_hz_h + h)}{\partial t} + \frac{\partial (cv_xv_h)}{\partial x} + \frac{\partial (cv_yv_h)}{\partial y} = 0 \]  

(8)

\[ \frac{\partial (\rho_m h v_x^2)}{\partial t} + \frac{\partial (\rho_m h v_x v_y)}{\partial x} + \frac{\partial (\rho_m h v_y^2)}{\partial y} + \frac{\partial (\rho_m - 1)}{\partial x} \frac{\partial h}{\partial x} + \frac{\partial (\rho_m - 1)}{\partial y} \frac{\partial h}{\partial y} = 0 \]  

(9)

\[ -\frac{\partial (c_{hv_x})}{\partial y} + r \sigma_{xv} + r \tau_{xv} = 0 \]  

(10)

where \( \rho_m = \rho \phi + \rho_s (1 - \phi) \) is the density of the mixture, \( \rho \) is the density of the water and \( \rho_s \) the density of the solid material in the bed, \( c \) the depth-averaged concentration, \( c_{hv} \) the maximum packing concentration of the solid material in the bed, \( z \) the elevation of the mobile bed, \( \sigma_{xv}, \sigma_{yv}, \tau_{xv}, \tau_{yv} \) are the depth-averaged normal and tangential stress acting along the depth, \( \sigma_{xv}, \tau_{xv}, \sigma_{yv}, \tau_{yv} \) are the normal bed shear and stress components of the depth-averaged velocity vector \( \bar{v} \).

Finally:

\[ r = \sqrt{1 + \tan^2 \alpha_x + \tan^2 \alpha_y} \]  

(11)

is a parameter that takes into account the slope of the bed surface with respect to the horizontal plane \((x, y)\), and \( \alpha_x \) and \( \alpha_y \) are the angles that the plane tangent to the bed surface forms with the \( x \)- and \( y \)-axis, respectively.

As the number of unknowns are greater than the number of the equations, two closure relations have been defined. The former relationship, relevant to the bed shear stress, is derived from uniform-flow condition and the model adopt Bagold’s relation of dispersive stress (1954) integrated on the depth and modified by Takahashi (1978) on the basis of experimental data and starting from the assumption of infinite slope in uniform motion and uniform concentration distribution. The relation, written in a vectorial framework, results:

3.2 TRENT2D model

The TRENT2D code (see Rosati & Bagnudelli (2013a, b) for numerical details) has been developed at CUDAM (Centro Universitario per la Difesa dell’Ambiente Montano), University of Trento, and it has been used in several field cases (see e.g. Rosati et al. 2015). The relevant mathematical model is based on the assumption of a variable-concentration mixture of sediments (solid phase) and water (Newtonian incompressible liquid phase), in which there is no-phase lag in module and direction between the depth-averaged velocity vectors of the solid and the liquid phase (Armanini et al. 2009). For this reason, this model can be defined as “quasi-two-phase model”,
\[
\frac{\tau_b}{\rho_w} = F(\|v\|, h)v
\]  
\text{(12)}

where

\[
F(\|v\|, h) = \frac{25}{4} \frac{\rho_c}{\rho_v} \sin \phi_d \frac{\lambda^2}{Y^2} \|v\|
\]  
\text{(13)}

and

\[
\lambda = \left[ \left( \frac{c_b}{c} \right)^{1/3} - 1 \right]^{-1}
\]  
\text{(14)}

is the linear concentration, \(\phi_d\) is the dynamic friction angle of the material, \(Y = h/(d\sqrt{a})\) where \(a\) is a constant assumed by Bagnold (1954) equal to 0.042 and estimated by Takahashi (1978) equal to 0.32, while \(d\) is the grain size of the solid phase. In case of non-uniform distribution, experimental evidence (A. Armanini, personal communication) suggests that a significant value for \(d\) is the mean diameter. A more detailed definition is actually not necessary because numerical simulations shows the results are only weakly dependent on the submergence parameter \(Y\).

The latter closure relationship concerns the concentration, assuming the immediate adaptation hypothesis, that is the sediment concentration is equal to the transport capacity that is to the concentration of a debris flow flowing in uniform conditions with the actual local and instantaneous velocity and depth (Fraccarollo & Capart 2002). The expression, taking into account the Meyer-Peter and Müller (1948) bed load formula, results:

\[
c = c_i \frac{\beta h^2}{gh}
\]  
\text{(15)}

where \(\beta\) is a dimensionless transport parameter function of \(d\), that can be estimated from laboratory data, or from the Takahashi closure for the concentration:

\[
c = \frac{i}{\Delta \tan \phi_d - i}
\]  
\text{(16)}

In this last case, assuming a 1D uniform flow, it follows that:

\[
\beta = \frac{c}{c_b} \frac{\lambda^2 K_T}{(1 + c\Delta)i}
\]  
\text{(17)}

where

\[
K_T = \frac{25}{4} \frac{\rho_c}{\rho_v} \sin \phi_d \frac{1}{Y^2}
\]  
\text{(18)}

\[
\Delta = \frac{(\rho_c - \rho_v)}{\rho_v}
\]  
\text{(19)}

and \(i\) the slope of the bed on the upstream boundary section.

4 DESCRIPTION OF THE STUDY AREA

The Pellaud catchment basin is located in the Granian Alps, in Aosta Valley Region (Northwestern Italy). The slope object of study is placed on the orographic left of Dora di Rhêmes river. It is close to the ridge which subdivides Rhêmes Valley and Valgrisanche Valley and is periodically subjected to debris flow phenomena.

The area is characterized by a wide valley, on top of which Becca di Fos (3459 m asl), Grand Rousse South (3555 m asl) and Grand Rousse North (3607 m asl) culminate. This area is called Pellaud, by the name of the stream which takes origin on this slope. Pellaud basin extents from 1810 m asl, when the stream flows into the main channel (Fig. 1).

The geological and geomorphological setting of the area is typical of the Northwestern Italian Alps, with metamorphic rocks, in particular gneiss and micaschists, belonging to the Median Pennidic Gran San Bernardo system.

The slope, characterized both by sediments and rock outcrops, reaches steepness greater than 30°.

The morphology of the area derives from the modelling action of glaciers, nowadays almost retreated, and from erosive action of surface water. In fact, on these slopes, the Pellaud stream origins from an almost retreated glacier, at 2600 m asl. For these reasons the stream flow can be considered as permanent. Furthermore, in the higher part of the catchment, the Pellaud stream is composed by branches.

Another secondary branch can be observed on the orographic right of the Pellaud stream, just in the proximal part of the alluvial fan (Fig. 1). Nevertheless, this branch has not been active since 2003, providing that, at present, the stream flows only in the principal branch. From a pluviometric point of view, the Rhêmes Valley is characterized by intense rainfall.

Moreover, the area is affected by the presence of not negligible unconsolidated debris volumes, derived from the evolution of the permafrost in rock, causing rockfall phenomena and disintegration.
process originated by frost weathering. The match between this abundant resource of unconsolidated material and intense rainfalls has periodically caused debris flow phenomena, particularly in the last 15 years. Volumes of debris involved have been estimated in the range between $5000-15000 \text{ m}^3$, and the origin has been rightly attributed to crumbling of rocks and glacier retreat.

5 NUMERICAL MODEL SETTINGS AND CALIBRATION CRITERIA

As already mentioned, due to the high steepness that characterizes the upper part of the Pellaud basin, a one-phase numerical code has been applied for runout analysis in the portion of the slope where the Takahashi's condition (1978) is not satisfied. The two-phase numerical code TRENT2D has been employed as soon as the criterion already mentioned is respected. The following analysis concerns a scenario with $5000 \text{ m}^3$ of debris involved.

As regards the transitional zone, in this area the outputs of the one-phase rigid bed code RASH$^{3D}$ have been used to obtain the input of the TRENT2D code.

This assumption has been justified considering that the value of the rheological parameters involved in the one-phase code analysis have been derived from numerous back analysis on real cases. Furthermore, due to the presence of rock outcrops in the upper part of the flow, no erosion could be expected in this area.

According to these facts, the spatial distribution of the material has been estimated as surveyed on site, and the rheological parameters adopted have been $\tan \phi = 0.05$, $\zeta = 1000 \text{ m/s}^2$, thus using a Voellmy rheology. Considering time intervals of about 80 s, at 160 s the flowing mass reaches the transitional zone with a velocity of about 9 m/s and height of 0.2 m (Fig. 2), determining a peak discharge of the debris flow of $32 \text{ m}^3/\text{s}$ in the flood hydrograph in this section. Due to the steepness of the upper part of the basin, the velocity is high, reaching about 12 m/s at 80 s, in correspondence of the intersection of the two main branches (Fig. 1).

Focusing on the transitional section velocity value progressively decreases in time to about 2 m/s. At about 800 s all the mass has passed through this section. The inflow flood hydrograph is obtained by RASH$^{3D}$ output data, has been used as input in TRENT2D, together with the local slope in this section, of 16.5°. The parameter $\beta$ derived from Equation (17) is of 0.725.

The physical parameters required used are:

- Dynamic friction angle of the material $\phi_d$: 38°
- Maximum packing concentration of the solid material in the bed $c_b$: 0.65
- Bulk density of the solid fraction $\gamma_s$: 26 kN/m$^3$
- Bulk density of the fluid fraction $\gamma_w$: 9.81 kN/m$^3$
- Sommergence parameter $Y$: 20.

Maintaining the same time interval used for RASH$^{3D}$ analysis, the TRENT2D simulation has been carried out for 2000s.
The results obtained are presenting and discussing in the following section.

6 ANALYSIS OF TRENT2D RESULTS

The obtained results are here introduced in terms of flow velocity, depth, evolution of morphology in time and space. Concerning the spatial distribution, they could be subdivided in: motion in the alluvial fan and its evolution in the Dora di Rhêmes river.

In the alluvial fan the flow is strongly channelized inside the riverbed of the Pellaud stream, even if a mass flows with a negligible height, less than 10 cm, in the orographic right, just in proximity of the no more active secondary branch (Fig. 1). Analyzing the maximum height, the absolute maximum value, of about 1.2 m, can be observed at 160 s, localized in the body of the flow, with homogeneous value inside all mass. Maximum height gradually reduces during the simulation until 0.51 m at 2000 s, and the area of the maximum value progressively moves to the front of the moving mass since it reaches the confluence zone with Dora di Rhêmes river. In fact, for the morphological configuration of this area, the moving mass deposits there, thus creating a natural barrier for the flow (Fig. 3). Except for the last consideration, the same trend can be observed focusing on the maximum velocity value, which decreases in time from about 3 m/s to 160 s, just in correspondence of the point of maximum height, to 0.98 m/s at the end of the simulation. The position of its maximum progressively moves from the body to the front of the moving mass over time.

Analyzing the erosion and deposition process, it could be noticed that both the positions of the maximum erosion and deposition during time remain the same and they are both placed in correspondence of the initial section, where, in fact, solid concentration and velocity, due to the steepness of the initial part, reach the highest values (Fig. 3). In fact, it could be remarked that the concentration of solid is correlated with slope in agreement to Takahashi's theory (1978, 2007, 2009); so that higher concentration produces more mass which can be deposited. The value of erosion and deposition are restraint between -4.43 to 3.15 m, and these maximum vales are recorded at the time 2000s (Fig. 3).

As already noticed, an area of deposit is localized in the confluence zone between Pellaud stream and Dora di Rhêmes river, just in the same area where a deposition process has been observed during the recent debris flows event of August 2015.

The entire flowing process was however simulated with the one-phase RASH3D model as well. A qualitative comparison between the two code results is given in Figure 4.

Taking into account that, with respect to RASH3D, TRENT2D allows volume changes along the runout path, due to the entrainment process.
a secondary channel of propagation in correspondence with the secondary branch already mentioned. Furthermore, when the flow reaches the confluence zone with Dora di Rhêmes river, it remains channelized, without invasion of adjacent areas.

7 CONCLUSIONS

Debris flow characteristics, depending on the dynamic interaction between solid and fluid phases during propagation, are a function of the percentage of soil, rock and water involved in the movement. Even if the numerical models inevitably rely on simplifying assumption, researchers are working since long time in frame of continuum mechanics to develop numerical tools able to simulate this phenomenon dynamics with increasing care. This is why, from one-phase models, there is a progressive converge to two-phase models.

In the present paper, the quasi-two-phase code TRENT2D, based on Takahashi’s theory (1978, 2007, 2009), has been used to simulate the dynamics of debris flow events in the Pellaud basin (Aosta Valley Region, Northwestern Italy).

Since in the upper part the Pellaud basin does not meet the conditions for validity of Takahashi’s theory (i.e. the angles of failing slopes of $\phi/2$ is exceeded), the one-phase code RASH 3D has been used to simulate the high-slope flow. Outputs of this model were used as inputs of TRENT2D as soon as the Takahashi’s condition are valid (i.e. gentle sloping).

Advantages of using TRENT2D concern the possibility to investigate some aspects of the flow dynamics in a more accurate and realistic way, that is: 1) the variation of solid concentration in the flowing mass during propagation, and 2) the entrainment of material along the runout path and changes in the morphology of the river-bed.

The application of TRENT2D to the Pellaud basin has evidenced two main aspects of the propagation process in this basin that reflect to a great extent the onsite conditions: 1) as soon the mass reaches the alluvial fan apex it splits in two branches, one that remains canalized along the main river track and moves in the intersection with the Dora di Rhêmes river, while the other that retraces an ancient runout path of debris flow events; 2) a depositional process occurs at the confluence between the Pellaud torrent and Dora di Rhêmes river.

In order to make a comparison between one-phase and quasi-two-phase results, the propagation process in the lower part of the slope and in the valley bottom was also simulated with the RASH 3D code.
Obtained results evidence a good agreement between the two codes in terms of general shape of the runout area, even if the spreading of the mass is larger in RASH$^{3D}$ than in TRENT2D. It has to be also remarked that no entrainment was considered in RASH$^{3D}$ analysis and this causes the underestimation of flowing depth with respect to TRENT2D results.

Finally, both the codes present advantages and disadvantages. TRENT2D can simulate a debris flow dynamics in a more accurate way but it cannot be applied to simulate the propagation process on a steep slope, while RASH$^{3D}$ does not present limitations in terms of topography but it introduces an important simplification modelling in a complex phenomenon like a debris flow as a one-phase movement.

ACKNOWLEDGMENTS

The Authors are grateful to Valle d’Aosta Region for provision of both digital terrain model and data of the Pellaud basin.

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